Funded PhD opportunity at University of Bath: Conceptual Denotational Semantics via Categorical Logic

Overview

My research revolves around applying category theory to computer science. In particular, I am interested in its applications to logic, programming languages and probability theory. I am looking for strong mathematicians or computer scientists with an interest in category theory to join my group at the University of Bath.

While you are not expected to be well-versed in category theory or programming language theory by the time you start the program, you are expected to have enough mathematical maturity so that you can learn these topics throughout your PhD.

If one of these topics interests you and you would like to hear/discuss more, please send me an email to phaa20@bath.ac.uk with a short (1~2 paragraph) introduction of yourself.

The department of Computer Science at the University of Bath has a very active, and expanding, theory group — four new lecturers have been hired over the last year. The position is fully funded for the entire duration of the program (3.5 years) and open to international as well as home students, meaning both tuition fees and a living stipend are covered.

First direction: Categorical logic and programming language theory

The field of denotational semantics studies the mathematical and compositional structures that govern programming languages and type theories. These structures are often used to prove properties about programs and formal systems. Examples of such properties range from reasoning at the program level, e.g. whether a particular program satisfies its specification, to language level, such as proving that no program can leak sensitive information or access invalid memory locations.

Recently, I have been interested in applications of fibered category theory to programming language theory [2, 4]. In particular, Grothendieck fibrations have proved themselves as effective abstractions for organizing the semantics of type theories [8], where the total category is fibered over a category of contexts, as well as a powerful way of constructing "logical relations" models [7, 9] — i.e. non standard models that encode useful global invariants of your syntax or semantics.

As a PhD student, you will advance type and programming language theories by developing techniques in domains such as fibered category theory, 2-category theory, universal algebra and categorical logic. These developments will be motivated by applications to domains such as substructural types a la Rust and probabilistic/differentiable computations. Depending on your interests, you can choose focus on either the computer science or mathematical aspects of the project.

Second direction: Categorical probability theory

As a PhD student you will work on the emerging field of categorical probability theory [6]. This field aims at reformulating many concepts from probability theory using the language of category theory. The goal is to abstract away from "implementation" details imposed by the measure theoretic axiomatization of probability in favor of more conceptual definitions that scale beyond measure theory.

Markov categories, which serve as an abstraction of Markov kernels, are one of the main objects of study in this field. While this definition has proved itself as robust in terms of accommodating useful aspects of probabilistic reasoning, such as conditional independence, Bayesian conditioning, etc. it can't accommodate the functional analytic account of probability theory [5]. This is unfortunate because functional analysis has been very successful in reasoning about stochastic processes by modeling them as linear operators between Banach spaces.

In recent work, I have proposed an extension of Markov categories that handles linear algebraic approaches to probability theory [3]. I achieved this by pairing Markov categories with a symmetric monoidal closed category (SMCC) and a lax monoidal functor between them. SMCCs have deep connections to linear algebra and logic, through the categorical semantics of substructural logics [10].

While my first results are promising, there is still much theory to be developed. Some natural questions are: how can we axiomatize aspects from probability theory such as conditional expectation or Bayesian inversion in this setting? Are there useful programming abstractions that can be justified by this formalism [1]?

Bibliography

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[9] Shin-ya Katsumata, Tetsuya Sato, and Tarmo Uustalu. 2018. Codensity Lifting of Monads and its Dual. *Logical Methods in Computer Science* (2018).

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